Property of Completeness

Every decimal number represents a real number, and every real number can be represented as a decimal.

The product and quotient properties of square roots can be used with a table of square roots to approximate irrational square roots if you don't have a calculator.

Example 2 Approximate each square root to the nearest hundredth.

a. $\sqrt{684}$ **b.** $\sqrt{0.8}$

Solution

a. $\sqrt{684} = \sqrt{2^2 \cdot 3^2 \cdot 19}$

$= \sqrt{2^2 \cdot 3^2} \cdot \sqrt{19}$

$= 6\sqrt{19}$

From the table: $\sqrt{19} \approx 4.359$

$6\sqrt{19} \approx 6(4.359) = 26.154$

$\therefore \sqrt{684} \approx 26.15$ ***Answer***

b. $\sqrt{0.8} = \dfrac{\sqrt{80}}{\sqrt{100}}$

$\approx \dfrac{8.944}{10} = 0.8944$

$\therefore \sqrt{0.8} \approx 0.89$ ***Answer***

Oral Exercises

State whether each number represents a rational or an irrational number.

1. $\sqrt{17}$ **2.** $\sqrt{49}$ **3.** $\sqrt{11}$ **4.** $\sqrt{1.21}$ **5.** $5 + \sqrt{2}$

6. $(\sqrt{2})^4$ **7.** $\sqrt{3} - \sqrt{3}$ **8.** 7π **9.** $2.\overline{91}$ **10.** $1.23456789\ldots$

Simplify.

11. $\sqrt{50}$ **12.** $\sqrt{150}$ **13.** $\sqrt{98}$ **14.** $\sqrt{128}$ **15.** $\sqrt{220}$

Approximate each square root to the nearest tenth. Use your calculator or the table at the back of the book.

16. $\sqrt{500}$ **17.** $\sqrt{1200}$ **18.** $\sqrt{2800}$ **19.** $\sqrt{4300}$ **20.** $\sqrt{6300}$

Written Exercises

Simplify.

A **1.** $\sqrt{63}$ **2.** $\sqrt{28}$ **3.** $\sqrt{98}$ **4.** $\sqrt{50}$ **5.** $\sqrt{75}$

6. $\sqrt{24}$ **7.** $\sqrt{256}$ **8.** $\sqrt{120}$ **9.** $2\sqrt{48}$ **10.** $6\sqrt{108}$

11. $5\sqrt{72}$ **12.** $9\sqrt{90}$ **13.** $\sqrt{529}$ **14.** $\sqrt{324}$ **15.** $6\sqrt{45}$

16. $14\sqrt{75}$ 17. $\sqrt{361}$ 18. $\sqrt{864}$ 19. $10\sqrt{125}$ 20. $3\sqrt{160}$
21. $\sqrt{192}$ 22. $\sqrt{432}$ 23. $5\sqrt{600}$ 24. $4\sqrt{363}$ 25. $6\sqrt{245}$
26. $5\sqrt{567}$ 27. $\sqrt{5625}$ 28. $\sqrt{9200}$ 29. $7\sqrt{1200}$ 30. $5\sqrt{2050}$

In Exercises 31–50, use your calculator or the table at the back of the book.

Approximate each square root to the nearest tenth.

B 31. $\sqrt{800}$ 32. $\sqrt{500}$ 33. $-\sqrt{700}$ 34. $-\sqrt{600}$
35. $-\sqrt{5900}$ 36. $-\sqrt{4800}$ 37. $\pm\sqrt{7800}$ 38. $\pm\sqrt{5600}$

Approximate each square root to the nearest hundredth.

39. $\sqrt{68}$ 40. $\sqrt{42}$ 41. $-\sqrt{0.5}$ 42. $-\sqrt{0.3}$
43. $\pm\sqrt{0.87}$ 44. $\pm\sqrt{0.73}$ 45. $-\sqrt{0.07}$ 46. $-\sqrt{0.08}$

Approximate each square root to the nearest whole number.

47. $\sqrt{150{,}000}$ 48. $\sqrt{240{,}000}$ 49. $\sqrt{420{,}000}$ 50. $\sqrt{580{,}000}$

Mixed Review Exercises

Find the indicated square roots.

1. $\sqrt{400}$ 2. $-\sqrt{169}$ 3. $\sqrt{\frac{25}{81}}$
4. $-\sqrt{\frac{49}{225}}$ 5. $\sqrt{176^2}$ 6. $\sqrt{\left(\frac{3}{8}\right)^2}$

Simplify.

7. $(17x)^2$ 8. $(3y^4z^9)^2$ 9. $(2x + 3y)^2$
10. $[15(a + 2)]^2$ 11. $(11a^4b^{11}c)^2$ 12. $(6z^3 + 5y^4)(6z^3 - 5y^4)$

Historical Note / π

The number π occurs naturally as the ratio of the circumference of a circle to its diameter. It is not possible to get an exact value for π since it is an irrational number.

The first known approximation (other than just using 3) was given in the Rhind mathematical papyrus as $(\frac{4}{3})^4$, or 3.1604 . . . . This was used until 240 B.C. when Archimedes calculated π to be between $\frac{223}{71}$ and $\frac{22}{7}$, or 3.14, to two decimal places. Four hundred years later this approximation was improved slightly to $\frac{377}{120}$, or 3.1416. In China, Tsu Ch'ung-chih gave a value for π of $\frac{355}{113}$, or 3.1415929 . . . , which is correct to six decimal places. Indian mathematicians used $\frac{62,832}{20,000}$, although this was later refined to $\frac{754}{240}$.